NOTATION

σ, stress; l, l_0, lengths of stretched specimen at time t and at time t = 0, respectively; l_r, length of a contracting specimen of elastic fluid after tension force has been removed; $l_r(\infty)$, length l_r at $t \to \infty$; d, diameter of a cylindrical specimen; α, elastic strain; $\varkappa$, deformation rate; F, tension force; $\sigma_0 = F/S_0$; S_0, cross-sectional area of a specimen at time t = 0; e_p, rate of irreversible deformation; η, maximum Newtonian viscosity; G_e, high-elasticity equilibrium modulus; $\theta = \eta/G_e$, relaxation time; E, activation energy of viscous flow; R, universal gas constant; T, some fixed temperature; T_k, some variable temperature; θ_k, $\varkappa_k$, t_k, relaxation time, deformation rate, and time at temperature T_k; and ε, full strain.

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TRANSIENT PROCESSES IN SHEAR FLOWS OF VISCOELASTIC FLUIDS. I. PROPAGATION OF A SHEAR WAVE

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A theoretical investigation is made of the initial stage of a transient process in the shear flow of a viscoelastic fluid having a relaxation-time spectrum.

A number of solutions are presently known for problems of transient shear flows of viscous and viscoelastic fluids. For example, flows of viscoelastic Maxwell and Oldroyd liquids having one relaxation time in a plane gap between parallel plates and a half-space with a plate set into motion impulsively were investigated in [1-6]. The nonsteady rotation of an infinite cylinder in a viscous fluid was analyzed in [7-9]. The development of fluid flow with relaxation and aftereffect of the Oldroyd type with an impulsively twisted cylinder is investigated in [10]. In [11] this same model was used to analyze freely damped oscillations of a cylinder by the method of a torsion pendulum. The results of such calculations are used to analyze nonsteady measurements in viscosimeters [12].

Because of the complexity of the molecular structure of polymer materials, their rheological behavior cannot be described by models of viscoelastic behavior with one relaxation time. For such media the character of the transient modes of deformation is determined to a considerable extent by the presence of a discrete relaxation spectrum. In this case a complete investigation of the dynamics of transient modes of deformation requires the distinguishing of the characteristic stages of flow, as well as their detailed qualitative and quantitative analysis, which are absent in [1-12].

As the rheological equation of state of the fluid we use Maxwell's generalized model with a relaxation-time spectrum reflecting the relaxation properties of polymers:

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$$\sigma = -PI + \tau, \quad \tau = \sum_{k=1}^{\infty} \tau_k, \tag{1}$$

$$\tau_k + \lambda_k F_{a00}\tau_k = 2\eta_k e, \quad k = 1,\ 2,\ \ldots\ \infty,$$

$$F_{a00}\tau_k = \mathcal{D}\tau_k/\mathcal{D}t + \vec{v}\cdot\nabla\tau_k + w\cdot\tau_k - \tau_k\cdot w + a(\tau_k\cdot e + e\cdot\tau_k).$$

Here σ is the stress tensor; P, isotropic pressure; τ, excess-stress tensor; I, unit tensor; $e = 1/2(\nabla\vec{V} + \nabla\vec{V}^t)$, deformation-velocity tensor; $\mathcal{D}/\mathcal{D}t$, substantial derivative; w, vorticity tensor. The following type of distribution of relaxation times and of the corresponding viscosities is used:

$$\lambda_k = \lambda/k^\alpha, \quad \eta_k = \eta_0/Z(\alpha)k^\alpha,$$

where λ is the greatest relaxation time in the system; η_0, initial Newtonian viscosity; $Z(\alpha)$, Riemann zeta function. Such a distribution of relaxation times λ_k and of the corresponding relaxation moduli $G_k = \eta_0/\lambda Z(\alpha)$ has a molecular-kinetic basis and is used in many rheological models of polymer liquids (see, e.g., [13, 14]). With an increase in k the contributions of the corresponding terms to the sum (1) decrease. In the theories of Rouse and Zimm $\lambda_k \sim \lambda/k^2$ and $\lambda_k \sim \lambda/k^{3/2}$. Henceforth we consider the case of $a = \pm 1$, when the operator F_{a00} coincides with the upper and lower convective derivatives, respectively.

The transient shear flow of a fluid in the gap between two coaxial cylinders (radii of the inner and outer cylinders R_1 and R_2) developing from a state of rest,

$$u(r,\ 0) = \tau_k(r,\ 0) = 0, \qquad R_1 \leqslant r \leqslant R_2, \tag{2}$$

is described by the system of equations

$$\rho\frac{\partial u}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau), \quad \tau = \sum_{k=1}^{\infty}\tau_k, \tag{3}$$

$$\tau_k + \lambda_k\frac{\partial\tau_k}{\partial t} = \eta_k r\frac{\partial}{\partial r}\left(\frac{u}{r}\right), \quad k = 1,\ 2,\ \ldots,\ \infty,$$

where ρ, u, and τ are the density, velocity, and shear stress in the fluid. We are analyzing the following three problems.

1. At the time t = 0 rotation with a constant velocity U is imparted to the outer cylinder. The inner cylinder is rigidly fixed:

$$u(t,\ R_1) = 0, \quad u(t,\ R_2) = U = \text{const},\ t > 0.$$

This problem permits an investigation of the wave dynamics of the development of shear disturbances in viscoelastic fluids.

2. At the time t = 0 a constant rotational velocity is imparted to the outer cylinder. The inner cylinder is connected to an elastic torsion:

$$u(t,\ R_2) = U, \quad R_1\frac{d\varphi}{dt} = u(t,\ R_1), \quad t > 0.$$

Here φ is the angular deviation of the inner cylinder from the equilibrium position, determined from the equation of motion of the inner cylinder with a length L and a moment of inertia I,

$$I\ddot{\varphi} - 2\pi R_1^2 L\tau(R_1,\ t) = -\varkappa\varphi(t), \quad t > 0,$$

where $\varkappa$ is the stiffness of the torsion. Such a statement gives an exact mathematical formulation to the problem of fluid flow in a viscosimeter with coaxial cylinders in the absence of end effects. A similar problem also arises in the description of fluid flow in a rotary instrument with a "cone-to-plane" working unit having a small gap angle [15, 16].

3. The outer cylinder is rigidly fixed, while a constant moment M of external forces, acting over a time interval t*, is applied to the inner cylinder at t = 0:

$$I\ddot{\varphi} - 2\pi R_1^2 L\tau(R_1,\ t) = M(t),$$

$$u(R_1,\ t) = u_1(t) = R_1\frac{d\varphi(t)}{dt}, \quad u(R_2,\ t) = 0,\ t > 0,$$

$$M(t)=\begin{cases} M=\text{const}, & 0\leqslant t\leqslant t^* \\ 0, & t>t^*. \end{cases}$$

This problem permits a description of the phenomenon of elastic recovery characteristic of viscoelastic fluids. It consists in a change in the direction of the rotation velocity of the inner cylinder after it is freed from the action of the external moment. In an inelastic fluid the inner cylinder slows down after unloading without changing the direction of the velocity. The method of application of a constant torsional moment to one of the measurement surfaces is widely used in rotary viscosimetry with coaxial cylinders [15, 16]. This makes it possible to combine the measurement of viscosity in the steady state with creep in the presteady state of shear. This method is especially suitable for measurements on instruments with considerable nonuniformity of the stress field. As noted in [15], the theory of the constant-moment method is inadequately developed.

These problems were solved numerically using a finite-difference approximation. A purely implicit, conservative, difference scheme was constructed by the integrointerpolation method [17]. Stability was demonstrated for it by the Fourier method. A quasiuniform spatial grid [18], which crowds together near the moving surfaces, is used to convey the initial stage of transient flow in more detail. At each time layer the system of difference equations was solved by the three-point trial-run method. The convergence of the numerical solution was tested on a series of problems having analytical solutions, as well as by comparing results obtained on crowding grids.

A conversion to dimensionless variables through the equations shows that the problems

$$r=R_1(1+\delta y),\quad t=\rho(R_2-R_1)^2\bar{t}/\eta_0,$$
$$u=V\bar{u},\quad \tau=\eta_0 V\bar{\tau}/(R_2-R_1)$$

under consideration contain two dimensionless parameters; the relative gap thickness $\delta=(R_2-R_1)/R_1$ and the elasticity number $El=\lambda\eta_0/\rho(R_2-R_1)^2$, characterizing the ratio of the relaxation time of the fluid to the time of development of the velocity profile in a plane gap containing the viscous fluid. In problems 2 and 3, where the dynamics of the inner cylinder is taken into account, there is an additional dimensionless parameter

$$D=\rho R_1^2 h^2 L/I=\frac{2\delta^2}{\pi[(1+\delta)^4-1]}\,\frac{I_0}{I},$$

which represents, with the accuracy of the multiplier, the ratio of the moment of inertia I_0 of the cylindrical layer of liquid with a thickness $h=R_2-R_1$ and a height L in the gap between the cylinders to the moment of inertia I of the inner cylinder. The rotation velocity U of the outer cylinder is taken as the characteristic flow velocity V in problems 1 and 2, while the rotation velocity of the inner cylinder in the established flow, $V=Mh/2\pi R_1^2 L\eta_0$, is taken in problem 3. Most of the calculations were made for values of $\delta=1$ and $D=1$. We mainly varied the quantities El and α characterizing the viscoelastic properties of the fluid for the rheological model under consideration. In place of the infinite series in (3) we took N terms and one purely viscous term with a "residual" viscosity:

$$\eta_{N+1}=\sum_{k=N+1}^{\infty}\eta_k=\eta_0\Big(1-\sum_{k=1}^{N}1/Z(\alpha)\,k^{\alpha}\Big).$$

The calculations were made for N from 1 to 24.

Let us consider the development of fluid flow in the gap when rotation is impulsively imparted to the outer cylinder while the inner one is rigidly fixed (problem 1). First we analyze the plane case for a qualitative estimate of the development of flow:

$$\rho\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left[\int_0^t\psi(t-t')\frac{\partial u(x,t)}{\partial x}\,dt'\right],\quad \psi(t)=\sum_{k=1}^{\infty}\frac{\eta_k}{\lambda_k}e^{-\frac{t}{\lambda_k}},\tag{4}$$

$$u(0,\ t)=V,\ t>0;\ u(x,\ 0)=0,\ x\geqslant 0,\ u(x,\ t)\to 0 \text{ as } x\to+\infty.$$

Problem (4) is solved using a Laplace time transformation (for example, see the solution of such problems for another type of function $\psi(t)$ in [19, 20]),

$$p\rho\hat{u}=\hat{\psi}(p)\frac{d^2u}{dx^2},\ \hat{u}(0)=\frac{V}{p},\ \hat{u}\to 0 \text{ as } x\to+\infty,$$

where $\hat{u}=\int\limits_0^\infty u\exp(-pt)\,dt$. From this we obtain the transform of the field of velocities

$$\hat{u}=\frac{V}{p}\exp\left(-\sqrt{\frac{p\rho}{\hat{\psi}(p)}}\,x\right) \tag{5}$$

and of shear stresses acting on the moving plane

$$\hat{\tau}(0)=\sqrt{\hat{\psi}(p)\rho/p}\,V. \tag{6}$$

Since $\hat{\psi}(p)/\eta_0$ depends only on λp, it follows from (6) that the quantity $\tau(t)/(\eta_0\rho V^2/\lambda)^{1/2}$ depends only on the ratio t/λ. Let us consider the development of flow at $t \ll \lambda$. In this case $|\lambda p| \gg 1$. The asymptotic behavior of the function $\hat{\psi}(p)$ as $|\lambda p|\to\infty$, found just as in [21, 22], is given by the expression

$$\hat{\psi}(p)\approx\eta_0\frac{\pi}{Z(\alpha)\alpha\sin\left[\frac{\pi}{\alpha}\right](\lambda p)^{1-\frac{1}{\alpha}}}. \tag{7}$$

After substituting this relation into (5) and converting to the inverse transform, we find

$$\psi(t)\approx\frac{\eta_0}{\lambda}\frac{\pi}{Z(\alpha)\alpha\sin\frac{\pi}{\alpha}\Gamma\left(1-\frac{1}{\alpha}\right)}\left(\frac{\lambda}{t}\right)^{\frac{1}{\alpha}},$$

$$u(x,\ t)\approx\frac{V}{2\pi i}\int\limits_{c-i\infty}^{c+i\infty}\frac{\exp\left(pt-c_\alpha xp^{1-\frac{1}{2\alpha}}\right)}{p}\,dp, \tag{8}$$

where

$$c_\alpha=\left[\rho\alpha Z(\alpha)\sin\left(\frac{\pi}{\alpha}\right)\lambda^{1-\frac{1}{\alpha}}/\eta_0\pi\right]^{\frac{1}{2}}.$$

Making the substitution $p\to z/t$ in the integral of (8), we obtain

$$u(x,\ t)\approx Vf(\xi,\ \alpha), \tag{9}$$

where

$$\xi=\frac{c_\alpha x}{t^{1-\frac{1}{2\alpha}}}=x\sqrt{\frac{\rho\alpha\sin(\pi/\alpha)Z(\alpha)}{\pi\eta_0\lambda}}\left(\frac{\lambda}{t}\right)^{1-\frac{1}{2\alpha}};$$

$$f(\xi,\ \alpha)=\frac{1}{2\pi i}\int\limits_{c-i\infty}^{c+i\infty}\frac{\exp\left(z-\xi z^{1-\frac{1}{2\alpha}}\right)}{z}\,dz.$$

Thus, the velocity distribution in the fluid at $t \ll \alpha$ is self-similar and depends only on the quantity ξ. From (9) with $t \ll \lambda$ we find the following expression for the coordinate of the front of the shear wave:

$$x_\varphi=\xi_\varphi\left(\frac{t}{\lambda}\right)^{1-\frac{1}{2\alpha}}\sqrt{\frac{\pi\eta_0\lambda}{\rho\alpha\sin\left(\frac{\pi}{\alpha}\right)Z(\alpha)}}.$$

This equation shows that the propagation of a shear wave slows with an increase both in α with a fixed t/λ and in λ with a fixed t. As $\alpha \to \infty$, $f(\xi, \alpha) \simeq H(1 - \xi)$, where $H(\xi)$ is the Heaviside function, equal to unity for $\xi \geq 0$ and zero for $\xi < 0$; $\xi \simeq (x/t)\sqrt{\rho\lambda/\eta_0}$. This velocity distribution corresponds to the purely elastic case and represents a rectangular impulse which is transferred in the medium with a finite velocity $c = \sqrt{\eta_0/\lambda\rho}$. As $\alpha \to 1$ (it should be noted that such a transition is impossible in the rheological equation, since the series in (4) diverges for $\alpha = 1$),

$$f(\xi, \alpha) \approx \mathrm{erfc}(\xi/2), \quad \xi \approx x\sqrt{\frac{\rho}{\eta_0 t}}.$$

Such a velocity distribution corresponds to a purely viscous medium. The fluid flow involves the entire half-plane with a velocity decreasing monotonically to zero at infinity. For $t \gg \lambda$ ($|\lambda p| \ll 1$),

$$\hat{\psi}(p) \approx \eta_0, \quad \psi(t) \approx \eta_0 \delta(t), \tag{10}$$

where $\delta(t)$ is the Dirac delta function, i.e., we have purely viscous behavior of the fluid. Substituting this expression into (5) and converting to the inverse transform, we obtain the well-known self-similar velocity distribution in a viscous fluid [23]

$$u(x, t) \approx V\,\mathrm{erfc}\left(\frac{\xi}{2}\right), \quad \xi = x\sqrt{\frac{\rho}{\eta_0 t}}.$$

Let us calculate the momentum imparted to a plate lying at a distance x from the moving plate. From Eq.(5) we find

$$-\frac{\hat{\tau}(p, x)}{p} = \frac{\rho V x}{p}\left[\frac{\hat{\psi}(p)}{p\rho x^2}\right]^{\frac{1}{2}} \exp\left(-\sqrt{\frac{p\rho x^2}{\hat{\psi}(p)}}\right). \tag{11}$$

We examine the asymptotic behavior of this expression at $t \ll \lambda$ and $t \gg \lambda$. At $t \ll \lambda$, substituting (7) into (11) and converting to the inverse transform, we obtain

$$-\int_0^t \tau(x, t)\,dt \approx \frac{\rho V x}{\xi}\Phi(\xi, \alpha),$$

where

$$\Phi(\xi, \alpha) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \frac{\exp\left(z - \xi z^{1-\frac{1}{2\alpha}}\right)}{z^{2-\frac{1}{2\alpha}}}\,dz.$$

For $\alpha \to \infty$, $\Phi(\xi, \alpha) \simeq (1 - \xi)H(1 - \xi)$, while for $\alpha \to 1$, $\Phi(\xi, \alpha) \simeq 2/\pi^{1/2} \exp(-\xi^2/4) - \xi\,\mathrm{erf}(\xi/2)$. For $t \gg \lambda$, with allowance for (10)

$$-\int_0^t \tau(x, t)\,dt \approx \frac{\rho V x}{\xi}\left[\frac{2}{\pi^{1/2}}\exp\left(-\frac{\xi^2}{4}\right) - \xi\,\mathrm{erfc}\left(\frac{\xi}{2}\right)\right].$$

Let us consider the development of shear stresses at a moving plate for short and long times. Substituting (7) into (6) for $t \ll \lambda$ and converting to the inverse transform, we find

$$\tau(0, t) \approx \left(\frac{\rho V^2 \eta_0}{\lambda}\right)^{\frac{1}{2}} \left(\frac{\pi}{\alpha \sin\frac{\pi}{\alpha} Z(\alpha)}\right)^{\frac{1}{2}} \frac{1}{\Gamma\left(1 - \frac{1}{2\alpha}\right)} \left(\frac{\lambda}{t}\right)^{\frac{1}{2\alpha}}. \tag{12}$$

For a fixed t/λ, $\tau(0, t)$ decreases with an increase in α. For $\alpha \to \infty$, $\tau(0, t) \simeq (\rho V^2 \eta_0/\lambda)^{1/2}$, which corresponds to a purely elastic medium. When $t \gg \lambda$,

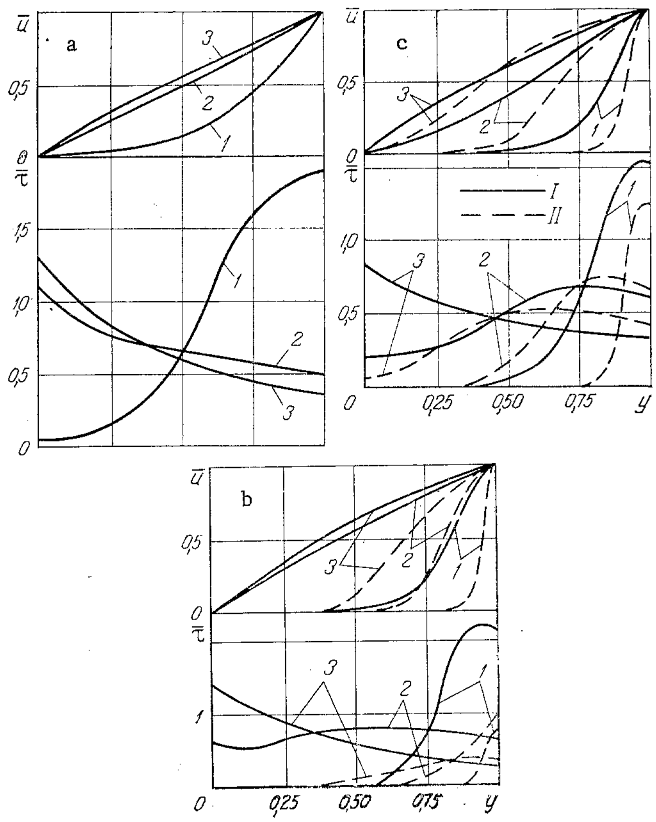

Fig. 1. Profiles of velocity and shear stresses in viscous and viscoelastic fluids for successive times: 1) $\bar{t}$ = 0.05; 2) 0.25; 3) 0.5; a) El = 0; b) α = 2.5; I) El = 0.1; II) 10; c) El = 1; I) α = 1.5; II) 3.

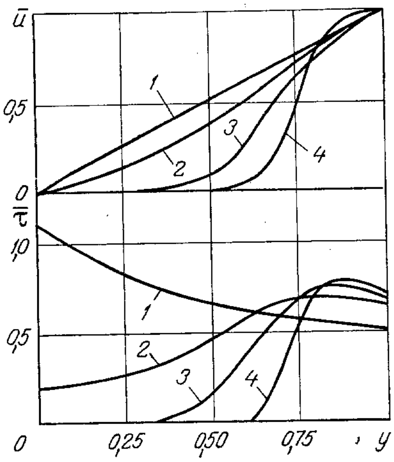

Fig. 2. Profiles of velocity and shear stresses in viscous and viscoelastic fluids for the time $\bar{t}$ = 0.25: 1) El = 0; 2) 1, α = 1.5; 3) El = 1, α = 3; 4) El = 1, α = 6.

$$\tau(0,\ t) \approx \left(\frac{\rho V^2 \eta_0}{\pi}\right)^{\frac{1}{2}} \frac{1}{t^{1/2}}. \tag{13}$$

The expressions derived allow one to estimate the main characteristics of the development of a shear wave. The time t_w of passage of the wave through the gap is determined by the relations

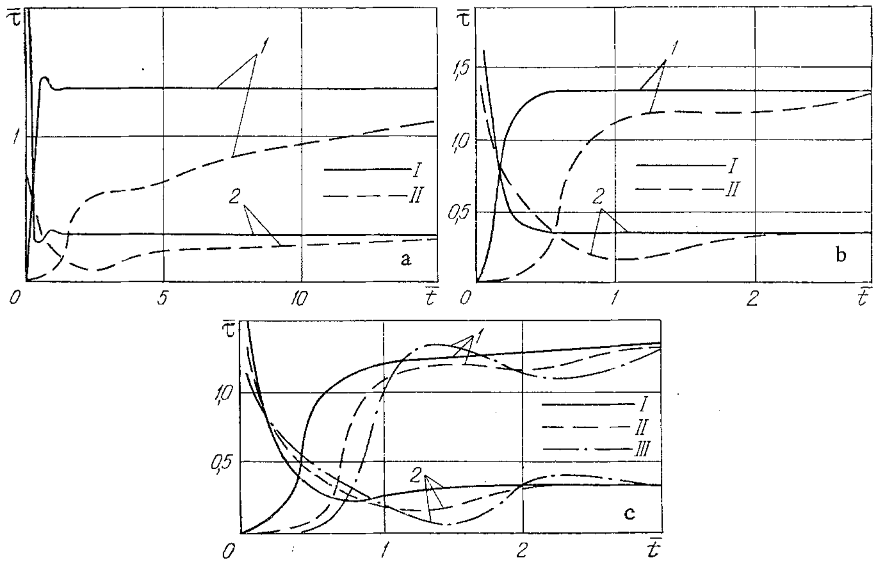

Fig. 3. Development of shear stresses at the inner (1) and outer (2) cylinders for viscous and viscoelastic fluids: a) I) El = 0; II) 1; α = 2.5; b) α = 2.5; I) El = 0.1; II) 10; c) El = 1; I) α = 1.5; II) 3; III) 4.5.

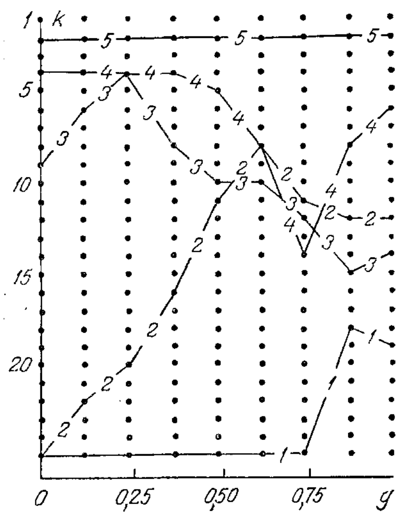

Fig. 4. Dynamics of the arrival of relaxators at the viscous regime (El = 1, α = 2.5; N = 24) for five successive times: 1) $\bar{t}$ = 0.05; 2) 0.25; 3) 0.5; 4) 0.75; 5) $\bar{t}$ = 2.

$$\frac{t_W}{\lambda} \sim \left(\frac{\alpha \sin \frac{\pi}{\alpha} Z(\alpha)}{\pi \mathrm{El}} \right)^{1/2\left(1-\frac{1}{2\alpha}\right)} \quad \text{for El} \gg 1, \; t_W \sim \frac{\rho h^2}{\eta_0} \quad \text{for El} \ll 1.$$

The increase in shear stresses at the outer cylinder is given by Eqs. (12) for $t \ll \lambda$ and (13) for $t \gg \alpha$. The momentum imparted to the inner cylinder over the time of propagation of the wave through the gap is

$$-\int_0^{t_W} \tau(h, t)\, dt \approx \rho V h \,\mathrm{const}(\alpha) \quad \text{for El} \gg 1, \tag{14}$$

$$-\int_0^{t_W} \tau(h, t)\, dt \approx \rho V h \quad \text{for El} \ll 1. \tag{15}$$

Let us turn to an analysis of the results of numerical calculations of the development of shear flow. Profiles of velocity and shear stress for viscous and viscoelastic fluids are shown in Figs. 1 and 2. The calculations for a viscous fluid (Fig. 1) lead to the well-known pattern of diffusional development of profiles of velocity and shear stress. At each point of the gap the velocity grows monotonically to a stationary value. The presence of elastic properties of the fluid imparts a wave character to the process of propagation of disturbance. The discontinuity of the velocity profile which occurs for Maxwell's model with one relaxation time [3-5] is absent for the model with a spectrum of relaxation times. The boundary of the region inside which the flow has developed by the given time has a rather blurred structure and gradually disappears. Such smoothing of the discontinuity also results from the use of Oldroyd's rheological model [3, 4], which allows for the time delay. In contrast to a viscous fluid, for a viscoelastic one the velocity at each point of the gap approaches the stationary value while undergoing damped oscillations about the limiting value. With an increase in the number El (Fig. 1) the elastic properties of the fluid grow and the wave character of the propagation of disturbances is displayed more strongly, as well as the oscillatory character of the establishment of stationary profiles of velocity and shear stress. An increase in the parameter α leads to a similar result (Figs. 1 and 2). This is explained by the weak influence of terms containing $k > 1$ in Eq. (3) and the approach of the rheological equation to Maxwell's model with one relaxation time. The development of shear stresses at the inner and outer cylinders is shown in Fig. 3. Because of the impulsive imparting of rotational velocity to the outer cylinder, the shear stresses at it, τ_2, grow without limit as $t \to 0$, which agrees with the estimates (12) and (13) obtained for viscoelastic and viscous fluids. As the numerical calculations showed, the presence of elastic properties of the fluid leads to a decrease in the initial values of the shear stresses at the outer cylinder.

The character of the arrival of various relaxators at the viscous regime for $El = 1$, $\alpha = 2.5$, and $N = 24$ can be seen from Fig. 4. The values of τ_k/τ_k^v were calculated for each term of the series (3) at nine equidistant spatial points, where $\tau_k^v = \eta_k r \frac{\partial}{\partial r}\left(\frac{u}{r}\right)$ is the shear stress for a viscous fluid with a viscosity η_k. As $t \to \infty$, $\tau_k/\tau_k^v \to 1$. In Fig. 4 each horizontal corresponds to one relaxator at different spatial points, and the regions in which $0.98 < \tau_k/\tau_k^v < 1.02$, which expand with time, are outlined for successive times. As seen from Fig. 4, one can be entirely confined to $N \approx 6$-10.

Thus, the above investigation shows the important influence of the elastic properties of a fluid and the relaxation time distribution on the characteristics of the initial stage of transient shear flow.

An investigation of transient modes of flow of a viscoelastic fluid with allowance for the dynamics of the inner cylinder will be made in later reports.

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